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# First Semester B.E. Degree Examination, Dec.2013/Jan.2014

## Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

**Note: 1.** ***Answer any FIVE full questions, choosing at least two from each part.***
**2.** ***Answer all objective type questions only in OMR sheet page 5 of the answer booklet.***
**3.** ***Answer to objective type questions on sheets other than OMR will not be valued.***



### PART – A

1 a. Choose the correct answers for the following :

i) If $y = \frac{x+2}{x+1}$, then $y_n$ is

A) $\frac{(-1)^n (n+1)!}{(x+1)^{n+1}}$ B) $\frac{(-1)^n n!}{(x+1)^{n+1}}$ C) $\frac{(-1)^n n!}{(x+1)^n}$ D) $\frac{(-1)^{n-1} n!}{(x+1)^{n+1}}$

ii) If $y = (ax+b)^m$ with m = n, then $y_n$ is

A) $n!\,a^n$ B) 0 C) $n!\,b^n$ D) $n!$

iii) The geometrical interpretation of Lagrange's mean value theorem is

A) $f'(c) = \frac{f(b)-f(a)}{b-a}$ B) $f'(c) = \frac{f(b)+f(a)}{b-a}$ C) $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$ D) none of these

iv) The Maclaurin's series expansion of $e^{-x}$ is

A) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} - - - -$ B) $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + - - - -$

C) $x - \frac{x^2}{2!} + \frac{x^3}{3!} - - - -$ D) $x + \frac{x^2}{2!} + \frac{x^3}{3!} + - - - -$ **(04 Marks)**

b. If $y = \sin \log (x^2 + 2x + 1)$, prove that $(x+1)^2 y_{n+2} + (2n+1)(x+1) y_n + (n^2+4) y_n = 0$. **(04 Marks)**

c. If x is positive, show that $x > \log(1+x) > x - \frac{x^2}{2}$. **(06 Marks)**

d. Using Maclourin's series, expand $\log(1+e^x)$ upto the terms containing $x^4$. **(06 Marks)**

2 a. Choose the correct answers for the following :

i) $\lim_{x \to \frac{\pi}{4}} \left( \frac{1 - \tan x}{\frac{\pi}{4} - x} \right)$ is equal to

A) 2 B) – 2 C) 1 D) – 1

ii) If $\phi$ be the angle between the tangent and radius vector at any point on the curve $r = f(\theta)$, then $\sin \phi$ is equal to

A) dr/ds B) $r\frac{d\theta}{ds}$ C) $r\frac{d\theta}{dr}$ D) ds/dr

iii) The rate at which the curve is bending called

A) radius of curvature B) curvature C) circle of curvature D) evolute

iv) The radius of curvature for polar curve $r = f(\theta)$ is given by

A) $\frac{(r^2 + r_1^2)^{3/2}}{r^2 + r_1^2 - rr_2}$ B) $\frac{(r^2 + r_1^2)^{3/2}}{r_1^2 + 2r^2 - rr_2}$ C) $\frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$ D) $\frac{(r^2 - r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$. **(04 Marks)**

b. Find the Pedal equation of the curve $r^m = a^m \cos m\theta$. **(04 Marks)**

c. Find the radius of curvature for the curve $y^2 = \frac{a^2(a-x)}{x}$, where the curve meets the x – axis. **(06 Marks)**

d. Evaluate $\lim_{x \to \infty} \left( \frac{ax+1}{ax-1} \right)^x$. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

3 a. Choose the correct answers for the following :

i) If $u = \log(x^2 + y^2 + z^2)$, then $\frac{\partial u}{\partial z}$ is

A) $\frac{2x}{x^2+y^2+z^2}$ B) $\frac{2y}{x^2+y^2+z^2}$ C) $\frac{2z}{x^2+y^2+z^2}$ D) $\frac{2z}{x^2+y^2-z^2}$

ii) If u =f(x, y) and y is a function x, then

A) $\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{dy}{dx}$ B) $\frac{\partial u}{\partial x} = \frac{du}{dx} + \frac{\partial u}{\partial y}\frac{dy}{dx}$

C) $\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial x}$ D) $\frac{\partial u}{\partial x} = \frac{du}{dx} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial x}$

iii) If $r = \frac{\partial^2 f}{\partial x^2}$, $S = \frac{\partial^2 f}{\partial x \partial y}$ & $t = \frac{\partial^2 f}{\partial y^2}$, then the condition for the saddle point is

A) $rt - s^2 < 0$ B) $rt - s^2 = 0$ C) $rt - s^2 > 0$ D) $rt - s^2 \neq 0$

iv) If $u = x + y + z$, $v = y + z$, $z = z$, then $J\left(\frac{u, v, z}{x, y, z}\right)$ is equal to

A) 1 B) −1 C) 0 D) none of these

**(04 Marks)**

b. The focal length of a mirror is given by the formula $\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$. If equal errors, 'e' are made in the determination of u and v, show that the resulting error in f is $e\left(\frac{1}{u} + \frac{1}{v}\right)$. **(04 Marks)**

c. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$. **(06 Marks)**

d. If $x = u(1 - v)$, $y = uv$. Prove that $JJ' = 1$. **(06 Marks)**

4 a. Choose the correct answers for the following :

i) Directional derivative is maximum along

A) tangent to the surface B) normal to the surface

C) any unit vector D) coordinate axes

ii) If $r = |x_i + y_j + 2_k|$, then $\nabla r^n$ is

A) $nr^{n-1}$ B) $r^{n-1}$ C) $\nabla . \nabla r^n$ D) none of these

iii) If $f = 3x^2 - 3y^2 + 4z^2$, then curl (grad f) is

A) $4x - 6y + 8z$ B) $4x_i - 6y_j + 8z\,k$ C) $\vec{0}$ D) 3

iv) If the base vectors $e_1$ and $e_2$ are orthogonal then $| e_1 \times e_2 |$ is

A) 0 B) − 1 C) + 1 D) none of these

**(04 Marks)**

b. If $\vec{F} = (x + y + 1)i + j - (x + y)k$, show that $\vec{F} \cdot \text{crul}\,\vec{F} = 0$. **(04 Marks)**

c. Find constants 'a' and 'b' such that $\vec{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational. Also find a scalar function $\phi$ such that $\vec{F} = \nabla\phi$. **(06 Marks)**

d. Prove that a spherical coordinate system is orthogonal. **(06 Marks)**


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## PART – B

5 a. Choose the correct answers for the following :

i) $\int_0^{\pi} \sin^7 x \, dx$ is equal to

A) zero B) $\frac{32\pi}{35}$ C) $\frac{32}{35}$ D) $\frac{35\pi}{32}$

ii) The asymptote of $(2 - x)y^2 = x^3$ is

A) x = 2 B) y – axis C) x – axis D) none of these

iii) The area of the cordioid $r = a(1 - \cos\theta)$ is

A) $\frac{3\pi a^2}{2}$ B) $\frac{3\pi}{2}$ C) $\frac{a^2}{2}$ D) $\frac{3a^2}{2}$

iv) The entire length of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ is

A) 6a B) 3a C) 2a D) a. **(04 Marks)**

b. Evaluate $\int_0^{\pi} \log(1 + a\cos x)dx$ by differentiating under the integral sign. **(04 Marks)**

c. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$, using reduction formula. **(06 Marks)**

d. Find the volume of generated by the revolution of the curve $r = a(1 + \cos\theta)$ about the initial line. **(06 Marks)**

6 a. Choose the correct answers for the following :

i) The general solution of the differential equation dy/dx = (y/x) + tan (y/x) is

A) sin (y/x) = c B) sin (y/x) = cx C) cos(y/x) = cx D) cos (y/x) = c

ii) The family of straight lines passing through the origin is represented by the differential equation :

A) ydx + xdy = 0 B) xdy – ydx = 0 C) xdx + ydy = 0 D) ydy + xdx = 0

iii) The homogeneous differential equation Mdx + Ndy = 0 can be reduced to a differential equation, in which the variables are separated by the substitution

A) y = vx B) x + y = v C) xy = v D) x – y = v

iv) The equation y – 2x = c represents the orthogonal trajectories of the family

A) $y = ae^{-2x}$ B) $x^2 + 2y^2 = a$ C) xy = a D) x + 2y = a **(04 Marks)**

b. Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$. **(04 Marks)**

c. Solve (1 + xy) ydx + (1 – xy) xdy = 0. **(06 Marks)**

d. Find the orthogonal trajectory of the cordioids $r = a(1 - \cos\theta)$. **(06 Marks)**

7 a. Choose the correct answers for the following :

i) If every minor of order 'r' of a matrix A is zero, then rank of A is
A) greater than r  B) equal r  C) less than or equal to r  D) less than r.

ii) The trivial solution for the given system of equations x + 2y + 3z = 0, 3x + 4y + 4z = 0, 7x + 10 y + 12z = 0 is
A) (1, 1, 1)  B) (1, 0, 0)  C) (0, 1, 0)  D) (0, 0, 0)

iii) Matrix has a value. This statement
A) is always true  B) depends upon the matrices  C) is false  D) none of these

iv) If A is singular and $\rho(A) = \rho(A : B)$ then the system has
A) unique solution  B) infinitely many solution  C) trivial solution  D) no solution. **(04 Marks)**

b. Using elementary transformations, find the rank of the matrix
$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}.$$ **(04 Marks)**

c. Show that the system x + y + z = 4 ; 2x + y – z = 1 ; x – y + 2z = 2 is consistent, solve the system. **(06 Marks)**

d. Apply Gauss – Jordan method to solve the system of equation :
2x + 5y + 7z = 52 ; 2x + y – z = 0; x + y + z = 9. **(06 Marks)**

8 a. Choose the correct answers for the following :

i) A square matrix A is called orthogonal, if
A) $A = A^L$  B) $A^T = A^{-1}$  C) $AA^{-1} = I$  D) none of these

ii) The eigen values of the matrix $\begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 2 \end{bmatrix}$ are
A) $1 \pm \sqrt{6}$  B) $1 \pm \sqrt{5}$  C) $\sqrt{5}$  D) 1

iii) The index and signature of the quadratic form $x_1^2 + 2X_2^2 - 3X_3^2$ are respectively
A) 2, [illegible]  B) 1, 2  C) 1, 1  D) none of these

iv) Two square matrices A and B are similar, if
A) A = B  B) $B = P^{-1}AP$  C) $A^T = B^T$  D) $A^{-1} = B^{-1}$. **(04 Marks)**

b. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12yz + 4zx - 8xy$ to the canonical form **(04 Marks)**

c. Determine the characteristic roots and eigen vectors of
$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$ **(06 Marks)**

d. Reduce the quadratic form $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_2x_3$ into sum of squares. **(06 Marks)**

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